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Gradients of seepage velocity model for contaminant transport prediction

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ABSTRACT

Models are commonly used to predict the contaminant transport. For example, hydraulic conductivity, porosity and gradient are parameters for seepage velocity model. The variation of each parameter needs to be considered for different grain size in order to determine accurate seepage velocity model. An experimental set up was used to determine hydraulic conductivity vis-à-vis permeability for sands of different porosities at different gradients, ranging from 1.15 to 15.00. The permeablities at gradient 1.15 are 0.466×10^{-11}, 1.054×10^{-11}, 1.175×10^{-11}, 1.690×10^{-11} and 3.293×10^{-11} m^2 for sands of porosities 0.250, 0.333, 0.364, 0.400 and 0.420 respectively, while the decline permeabilities obtained at higher gradient 15.0 are 0.334×10^{-11}, 0.942×10^{-11}, 1.023×10^{-11}, 1.550×10^{-11} and 2.684×10^{-11} m^2 for sands of porosities 0.250, 0.333, 0.364, 0.400 and 0.420 respectively. It was observed that pre-field test or models for fine grained sand and coarse grained sand cannot be subjected to the same gradients for high degree of prediction. Thus, in the model or pre-field test experiment, lower gradients (below 1.88) are more appropriate for coarse grained sand, while higher gradients (above 2.50) should be used for fine grained sand. This recommendation will be appropriate for the accurate prediction of contaminant transport seepage velocity in each case.

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Introduction

The flow of free water through soil is governed by Darcy's law, [1] demonstrated experimentally that for laminar flow in homogenous soil, the velocity of flow is proportional to hydraulic gradient, where constant of proportionality is hydraulic conductivity (or coefficient of permeability). The seepage velocity of fluid in porous media and permeability of the media are the major parameters needed in applying Darcy's law to environmental problems. These parameters were found to strongly dependent on the porosity of the media [2]. However, it was found that Darcy's law is not valid for extremely fine – grained soil, because of their low porosity [3–5]. It is very difficult to predict the exact range of the validity of Darcy's law in media with very low porosity. The best method to ascertain the range is to conduct experiments and determined the

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actual relationship between the velocity, v of the fluid and the hydraulic gradient (or driving force) as a function of porosity of the media. If this is done, the seepage velocity of fluid in wide range of media will be accurately determined in order to predict the travel time of contaminated or unwanted fluid through soils.

Models are commonly viewed as useful tools for understanding contaminant transport and determining future risk [6,7]. The degree of accuracy of the models to predict contaminant transport seepage has not been established [8,9]. A good model should have a high degree of capability to predict future exposure and risk. The purpose of this work is to determine the appropriate gradients required for sand of different sizes in modeling contaminant seepage control system.

Several investigations have published data show that hydraulic flow through saturated clays and fine grained sands deviates from Darcy's law [10–19]. Swartzendruber [19] suggested in one of his papers that classic law of Darcy be modified for certain types of porous media consisting of or containing clay. He observed that there are ranges of hydraulic gradient in which Darcy's law is not valid and also predicted the existence of threshold gradient apart from zero-gradient at which seepage velocity is zero. Also, in one of the recent saturated hydraulic conductivity determined on the field in Nigeria, it was observed that there is a disproportionate relationship between discharge flow rate and hydraulic gradient for finer particles [20].

The control of seepage involves reducing the flow, reducing the water pressure, or increasing the load that resists the water pressure [21]. This can be done by a proper selection of a protective filter of permeability. Permeability is the most physical property of a porous medium, which is a measure of the ability of a material to transmit fluid through it under a hydraulic gradient [22]. The application of Darcy's law enables permeability to be determined both in the laboratory and on the field [20,23,24].

Natural filters have been used as land fill liners to reduce the movement of contaminated fluid from a solid waste landfill (Waste water disposal) into subsurface groundwater and in filtering harmful particles from a fluid stream in industries [25–28]. The selection of a good filter as a protective layer (or a seepage control) depends on its ability to reduce the liquids (leachate) from waste to sub-surface groundwater.

Theoretical background

The volume flux is defined as:

$$q = \frac{Q}{A} \tag{1}$$

where, q = Darcy flux (ms^{-1}), Q = Volumetric flow rate (m^3s^{-1}), and A = Cross-sectional area (m^2)

The volume flux is the volumetric flow rate per unit area. Then, Darcy law is

$$q = -K\frac{dh}{dl} \tag{2}$$

where l = length of the sample (or medium), h = head constant (or hydraulic head).

Hydraulic conductivity, K is related to permeability, k by equation:

$$K = \frac{\rho g}{\mu}k \tag{3}$$

where K = hydraulic conductivity (ms^{-1}), k = permeability (m^2), μ = viscosity of the fluid (Nsm^{-2}), and ρ = density of the fluid, (kgm^{-3}) [23].

Seepage velocity

The volume flux q in Eq. (2) is not the actual velocity through the interstices of the soil [29]. It is fictitious velocity obtained by dividing the total discharge (Q) by the total cross-sectional area (A). The total cross-sectional area consists of not only the voids but also the solids. As flow can take place only through voids, the actual velocity through the voids is much greater than the volume flux (or discharge velocity). The actual velocity on the macroscopic scale is known as seepage velocity, v [29,30]. The seepage velocity v is equal to the volume flux divided by porosity and is given as:

$$v = \frac{Q}{A\varphi} = \frac{q}{\varphi} \tag{4}$$

where φ = porosity of the medium [30].

Methodology

Determination of porosity

Small quantity of each of the samples (A, B, C, D and E) which were to be used for determination of bulk volume were well dried in an electric oven at a temperature that is a little above the boiling point of water (120 °C) for about an hour, so that moisture was completely eliminated. The samples were allowed to cool down for two hours. 4 mL of the dried sand

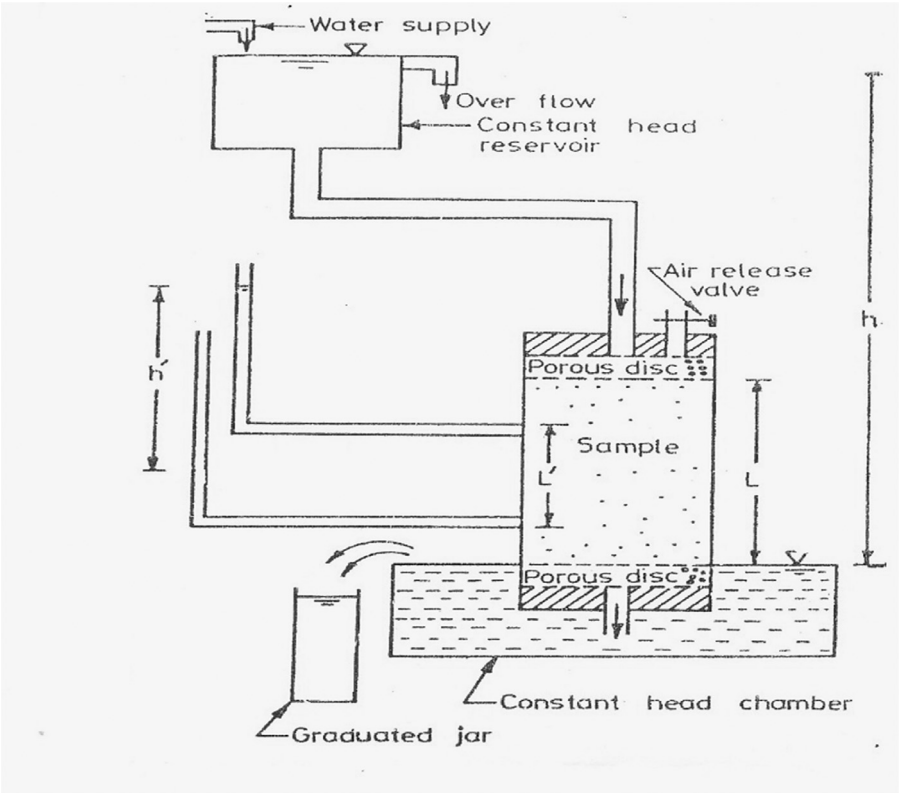

Fig. 1. Constant head Permeameter [29].

was measured using a 10 mL measuring cylinder. It was ensured that the measuring cylinder was tapped with a solid object and the sand inside re-arranged and compacted before the value of the volume was recorded. This is necessary in order to maintain steady volume. A similar measuring cylinder was half-filled with water and the volume was noted. The sand was then poured into the water and the final volume of the components of the cylinder (water and sand) was recorded. The displacement of the water yields the volume of grain, while pore volume is the difference between bulk volume and grain volume. Porosity is the ratio of pore volume to bulk volume.

Determination of volume flux and hydraulic conductivity at different hydraulic gradient (Constant head method)

The soil samples were soaked (saturated) overnight before being transferred to the transparent cylindrical tube of cross-sectional area 2.69×10^{-4} m^2. To ascertain uniform compaction throughout the samples, the screened end was blocked so as to prevent the water passage, the soaked sand was then poured into the water column up to the height h. After this, the material that was used to block the passage was removed. A continuous supply of water was fed through the sample and at height h, a hole was drilled, this enabled the height to be maintained as excess water was drained through it (Fig. 1). The water that passed through the sample for 60 s was collected with a beaker and the volume measured by measuring cylinder and steady-state flow has been maintained at a constant head. This procedure was repeated for all the samples.

By adoption of Darcy's fluid flow Eq. (2), length of sand, L is varied in order to obtain different hydraulic gradients, i. Measurements were made at gradients for each samples. The value of the volume flux, q at each hydraulic gradient was computed from the value discharge volumetric flow rate, Q, knowing that volume flux $q = Q/A$, where $A = \pi\,(d/2)^2$ and d is the diameter of the cylindrical tube used, given as 1.85×10^{-2} m. A plot was prepared between volume flux, q and hydraulic gradient, i for samples A–E. The slope of the respective lines gave the hydraulic conductivity of the medium concerned.

Determination of permeability

From the respective values of hydraulic conductivities, permeability of each of the sample was determined or computed by using the relation in Eq. (3).

Table 1
Relative values of permeability $k' \times 10^{-4}$ (m) at different hydraulic gradient.

Sample	gradient	porosity									
		1.15	1.25	1.50	1.88	2.50	3.00	3.75	5.00	7.50	15.00
A	0.250	0.466	0.466	0.466	0.465	0.456	0.436	0.415	0.385	0.355	0.334
B	0.333	1.054	1.054	1.054	1.044	1.023	1.013	0.722	0.983	0.962	0.942
C	0.364	1.175	1.175	1.155	1.135	1.104	1.094	1.074	1.064	1.045	1.023
D	0.400	1.690	1.662	1.655	1.631	1.611	1.591	1.581	1.570	1.550	1.550
E	0.420	3.293	3.273	3.262	3.232	3.019	2.786	2.746	2.715	2.695	2.684

Table 2
Deviation of relative permeabilities from the mean permeability for Sample A.

$K_r \times 10^{-4}$ (m/s)	$K_m \times 10^{-4}$	$k \times 10^{-4}$	$K_r - K_m$	$(K_r - K_m)^2$
0.46	0.315	0.466	0.145	0.021025
0.46	0.315	0.466	0.145	0.021025
0.46	0.315	0.466	0.145	0.021025
0.46	0.315	0.465	0.145	0.021025
0.459	0.315	0.465	0.144	0.020736
0.43	0.315	0.436	0.115	0.013225
0.41	0.315	0.415	0.095	0.009025
0.38	0.315	0.385	0.065	0.004225
0.35	0.315	0.355	0.035	0.001225
0.33	0.315	0.334	0.015	0.000225
total				0.132761

k = Permeability, K_r = Relative hydraulic conductivity, K_m = Mean hydraulic conductivity.
$(K_r - K_m)$ = Deviation from mean hydraulic conductivity, $(K_r - K_m)^2$ = Standard deviation.

Table 3
Deviation of relative permeabilities from the mean permeability for Sample B.

$K_r \times 10^{-4}$ (m/s)	$K_m \times 10^{-4}$	$k \times 10^{-4}$	$K_r - K_m$	$(K_r - K_m)^2$
1.04	0.9224	1.054	0.1176	0.01383
1.04	0.9224	1.054	0.1176	0.01383
1.04	0.9224	1.054	0.1176	0.01383
1.03	0.9224	1.044	0.1076	0.011578
1.01	0.9224	1.023	0.0876	0.007674
1	0.9224	1.013	0.0776	0.006022
0.71	0.9224	0.722	−0.2124	0.045114
0.97	0.9224	0.983	0.0476	0.002266
0.95	0.9224	0.962	0.0276	0.000762
0.93	0.9224	0.942	0.0076	5.78E−05
				0.114962

Results and discussion

Sample A and B are fine grained sand, sample C and D are moderate, while D and E are coarse grained sand. K_m is the mean value of the Hydraulic conductivity (HC), which is the actual value of the HC for a sample of a particular porosity. K_r is the relative HC of the sample as a result of variation of the hydraulic gradients ranging from 1.15 to 15.00. 1.15 and 15.00 are the minimum and maximum gradients used for the set up. Parameter k is the permeability obtained by computation from the HC by using Hubbert King Relation (Eq. (3)).

Table 1 shows the relative permeability at different gradient for samples A–E. There is no significant deviation $(K_r - K_m)$ from mean HC at lower gradients for samples A and B. This is shown in column 4 of Tables 2 and 3 for samples A and B respectively. However, the deviation is more significant as the gradient increasing. This deviation is more pronounced at higher gradients. Column 5 of the Tables 2 and 3 indicate the standard deviation (SD), which follows the same pattern as deviation.

Tables 4 and 5 show the deviation and SD for samples C and D respectively. The columns 4 and 5 indicate deviation and SD respectively in each case. The results show that both deviation and SD decreases as gradient increases.

Table 6 shows a significant deviation and SD at the lower gradients for sample E (coarse grained sand). This is obvious in column 4 and 5. In addition, the maximum variance (0.1847) obtained was found in sample E a shown in Table 7. This value is relatively higher when compare with others.

Table 4
Deviation of relative permeabilities from the mean permeability for Sample C.

$K_r \times 10^{-4}$ (m/s)	$K_m \times 10^{-4}$	$k \times 10^{-4}$	$K_r - K_m$	$(K_r - K_m)^2$
1.16	0.9966	1.175	0.1634	0.0267
1.16	0.9966	1.175	0.1634	0.0267
1.14	0.9966	1.155	0.1434	0.020564
1.12	0.9966	1.135	0.1234	0.015228
1.09	0.9966	1.104	0.0934	0.008724
1.08	0.9966	1.094	0.0834	0.006956
1.06	0.9966	1.074	0.0634	0.00402
1.05	0.9966	1.064	0.0534	0.002852
1.03	0.9966	1.045	0.0334	0.001116
1.01	0.9966	1.023	0.0134	0.00018
				0.113036

Table 5
Deviation of relative permeabilities from the mean permeability for Sample D.

$K_r \times 10^{-4}$ (m/s)	$K_m \times 10^{-4}$	$k \times 10^{-4}$	$K_r - K_m$	$(K_r - K_m)^2$
1.66	1.488	1.69	0.172	0.029584
1.64	1.488	1.662	0.152	0.023104
1.63	1.488	1.655	0.142	0.020164
1.61	1.488	1.631	0.122	0.014884
1.59	1.488	1.611	0.102	0.010404
1.57	1.488	1.591	0.082	0.006724
1.56	1.488	1.581	0.072	0.005184
1.55	1.488	1.57	0.062	0.003844
1.53	1.488	1.55	0.042	0.001764
1.5	1.488	1.519	0.012	0.000144
				0.1158

Table 6
Deviation of relative permeabilities from the mean permeability for Sample E.

$K_r \times 10^{-4}$ (m/s)	$K_m \times 10^{-4}$	$k \times 10^{-4}$	$K_r - K_m$	$(K_r - K_m)^2$
3.25	2.5847	3.293	0.6653	0.442624
3.23	2.5847	3.273	0.6453	0.416412
3.22	2.5847	3.262	0.6353	0.403606
3.19	2.5847	3.232	0.6053	0.366388
2.98	2.5847	3.019	0.3953	0.156262
2.75	2.5847	2.786	0.1653	0.027324
2.71	2.5847	2.786	0.1253	0.0157
2.68	2.5847	2.715	0.0953	0.009082
2.66	2.5847	2.695	0.0753	0.00567
2.65	2.5847	2.684	0.0653	0.004264
				1.847333

Table 7
Variation of relative permeabilities from the mean permeability for Samples A–E.

Sample	Porosity	Variance
A	0.25	0.0133
B	0.33	0.0115
C	0.364	0.0113
D	0.4	0.0116
E	0.42	0.1847

Taking the above results and observations into consideration, the pre-field test or models for fine-grained sand and coarse grained sand cannot be subjected to the same gradients for high degree of prediction. From the above results, it shows that lower gradients will be appropriate for coarse fine grained sand, while higher gradients will be appropriate for fine grained sand. This is true because this is the range of gradient where each of them shows significant deviation from the mean HC, and this must be taken into consideration whenever the model is being developed for high degree of prediction.

Conclusion

The results corroborate the fact that deviation from the actual (or mean) hydraulic conductivity, K_m increases with increase in hydraulic gradient and this implies that the gradient that Darcy law gives true hydraulic conductivity or permeability depends on the texture of the sand. The Darcy law is best fitted at lower gradient (below 1.88) for coarse grained sand, while it gives true hydraulic conductivity for fine grained sand at higher gradients (above 2.50). The experiment showed that the relative hydraulic conductivity vis-à-vis permeability decreases with increase in hydraulic gradient for sand. In addition the variation from actual permeability increases with increase in porosity of sand.

In addition, the pre-field test or models for fine grained sand and coarse grained sand cannot be subjected to the same gradients for high degree of prediction. Models in water pollution control in sand soil, lower gradients are recommended for coarse grained sand soil while higher gradients are recommended for fine grained sand soil. These recommendations will be suitable for accurate prediction of seepage of contaminant transport in each case.

References

[1] H. Darcy, Les Fountaines Publiquues De La Ville De Dijon, Victor Dalmont, Paris, 1856.
[2] E.N. List, N.H. Brooks, Lateral dispersion saturated porous media., J. Geophys. Res. 22 (2001) 25–31.
[3] F.G. Bell, Engineering Geology, Second ed., Butterworth-Heinemann, Elsevier print, Burlington, USA, 2007.
[4] O.O. Alabi, Validity of Darcy's law in laminar regime, Electron. J. Geotech. Eng. 16 bundle A (2011) 27–40 Oklahoma University, USA.
[5] O.O. Alabi, O.I Popoola, Adegoke J.A., Modification of Darcy's law for extremely fine-grained soils, Electron. J. Geotech. Eng. 17 bundle I (2012) 1305–1321 Oklahoma University, USA.
[6] Oreskes, N.R. Shrader-Frechette, K. Belitz, Verification, validation and confirmation of numerical models in the Earth sciences, Science 263 (1994) 641–646.
[7] ASTM, Standard Guide for Risk Based Corrective Action Applied at Petroleum Release Sites, American society for Testing and Materials, 1995 ASTM Designation E 1739095.
[8] C.T. Miller, W.G. Gray, Hydrogeological research: just getting started, Ground Water 40 (3) (2002) 224–231.
[9] J.R. Eggleston, S.A. Rojstaezer, Can we predict substance mass transport? Environ. Sci. Technol. 34 (2000) 4010–4017.
[10] D.E. Desaulniers, J.A. Cherry, Origin and movement of grand water and major ions in a thick deposit of Champlain sea clay near Montreal, Can. Geotech. J. 26 (1989) 80–89.
[11] E. Douglas, N.R. Yong, Validity of Darcy's Law for one dimensional, unsaturated flow in kaolinite – water – NaCl systems, Trans. Am. Soc. Agric. Eng. 24 (3) (1981) 657–662.
[12] J. Hoeks, H. Glas, J. Hofkamp, A.H. Ryhiner, Bentonite liner for isolation of waste disposal sites, Waste Manage. Res. 5 (1987) 93–105.
[13] K.T. Law, C.F. Lee, Initial gradient in a dense glacial till proceedings, in: 10th International Conference on Soil Mechanics and Foundation Engineering, Stockholm, Sweden, 1981, pp. 441–446.
[14] P.F. Low, Nature and properties of water in montmorillonite-water systems, Soil Sci. Soc. Am. J. 43 (1979) 651–658.
[15] R.J. Miller, P.F. Low, Threshold gradient for water flow in clay systems, Soil Sci. Soc. Am. Proc. 27 (6) (1963) 605–609.
[16] A.J. Silva, J.R. Hetherman, D.I. Calnan, Low gradient permeabilities testing of fine-grained marine sediments, in: T.F. Zimmie, C.O. Riggs (Eds.), Permeability and Groundwater Contamination Transport, 1981, pp. 121–136.
[17] R. Sri Ranjan, R.W. Gillham, Non – Linear Electro – Osmotic and Hydraulic Flow Phenomena in Saturated Clays, Canadian society of Agric. Engr., 1991 Paper No. 91-105.
[18] W.E. Stevens, The Flow of Non-Newtonian Fluid in Circular Conduits Ph.D Thesis, University of Utah, 1953.
[19] D. Swartzendruber, Non- Darcy flow liquid- saturated porous media, J. Geophys. Res. 67 (1962) 2508–2510.
[20] O.A. Ehigiator, B.U. Anyata, Saturated hydraulic conductivity in a tropical Afisol in South- West Nigeria, Env. Res. J. 2 (2) (2008) 70–75.
[21] G.B. Sowers, G.F. Sowers, in: Introductory Soil Mechanics and Foundation, Third ed., Macmillan Publishing Co. Inc, New York, 1970, pp. 85–87.
[22] J.K. Sherwani, in: Elements of Water Supply and Waste Water Disposal, Second ed., John-Willey and sons, Inc., 1978, p. 122.
[23] P.A. Domenico, F.W. Schwartz, Physical and Chemical Hydrogeology, John Willey and Sons Inc., New York, 2000.
[24] O.I. Popoola, J.A. Adegoke, O.O. Alabi, Modelling the flow of water in stratified layers of sand, J. Nig. Ass. Math. Phys. 18 (2011) 119–130.
[25] D.E. Daniel, R. Benson, Summary review of construction quality control for compacted soil lines, in: R. Bonaparte (Ed.), Waste Containment System Construction, Regulation and Performance, ASCE, New York, 1990, pp. 175–189.
[26] C.H. Benson, J. Trast, Hydraulic conductivity of thirteen compacted clays, Clay Clay Min. 43 (6) (1995) 669.
[27] R.K. Rowel, R.M. Quigley, J.R. Booker, in: Clay Barrier System for Waste Disposal Facilities, Chapman and Halpl, London, 1995, p. 390.
[28] F.K. Boadu, Hydraulic conductivity of soils from grain size distribution: new models, J. Geotech. Geoenviron. Eng. 126 (8) (2000) 739–746 ASCE.
[29] K.R. Arora, Soil Mechanics and Foundation Engineering, (Geotechnical 7th Engineering) ed., Standard publishers distributors, Delhi, 2009.
[30] J. Bear, V. Arnold, Modelling Groundwater flow and Pollution, Reidel Publishing Company, 1990.